7. Solving Linear Inequalities and Compound Inequalities

Steps for solving linear inequalities are very similar to the steps for solving linear equations. The big differences are multiplying and dividing a constant on the inequalities and expressing the solution set. **However, if you want to practice with solving linear equations, you can refer to the previous topic. (Topic 6)** This handout will show some examples on how to solve linear inequalities and compound inequalities and how to express the solution sets of inequalities.

Solve Linear Inequalities

Example (1): $3x+8>6$

Solution:

$$3x+8-8>6-8$$
$$3x>-2$$

Subtract 8 on each side

$$\frac{3}{3}x>\frac{-2}{3}$$
$$x>\frac{-2}{3}$$

Divide 3 on each side. **Do not reverse** the inequality symbol.

The solution set is $\left\{x \middle| x>\frac{-2}{3}\right\}$

Place the solution set in the set-builder notation

Example (2): $3x-2\geq 5x+13$

Solution:

$$3x-2+2\geq 5x+13+2$$

Add 2 on each side

$$3x\geq 5x+15$$

Simplify

$$3x-5x\geq 5x-5x+15$$

Subtract 5x on each side

$$-2x\geq 15$$

Simplify

$$\frac{-2x}{-2}\leq\frac{15}{-2}$$
$$x\leq-\frac{15}{2}$$

Divide -2 on each side; **reverse** the inequality symbol (when divide or multiply a negative number)

The solution set is $\left\{x \middle| x\leq-\frac{15}{2}\right\}$

Place the solution set in the set-builder notation.

Example (3): $6(3+4x)-2<20$

Solution:

$18+24x-2<20$ — Remove the parenthesis by multiplying 6 to 3 and 4x.

$24x+16<20$ — Simplify

$24x+16-16<20-16$ — Subtract 16 on each side

$24x<4$ — Simplify

$\frac{24x}{24}<\frac{4}{24}$ — Divide 24 on each side. **Do not reverse** the inequality symbol.

$x<\frac{1}{6}$ — Simplify

The solution set is $\left\{x \,\middle|\, x<\frac{1}{6}\right\}$ — Place the solution set in the set-builder notation

Example (4): $\frac{1}{2}(w-3)-(2-w)\le 1$

Solution:

$(2)\frac{1}{2}(w-3)-(2)(2-w)\le(2)1$ — Multiply 2 on **each term** to simplify the inequality

$(w-3)-2(2-w)\le 2$ — Simplify

$w-3-4+2w\le 2$ — Remove parenthesis. Multiply -2 to $(2-w)$

$3w-7\le 2$ — Simplify

$3w-7+7\le 2+7$ — Add 7 on each side

$3w\le 9$ — Simplify

$w\le 3$ — Divide 3 on each side. **Do not reverse** the inequality symbol.

The solution set is $\{w \mid w\le 3\}$ — Place the solution set in the set-builder notation

Example (5): $\dfrac{5z-4}{5} > \dfrac{2+5z}{3}$

Solution:

$(15)\dfrac{5z-4}{5} > (15)\dfrac{2+5z}{3}$ Find LCD=15. Multiply 15 to each term

$3(5z-4) > 5(2+5z)$ Simplify

$15z-12 > 10+25z$ Distribute property to remove the parenthesis

$15z-12+12 > 10+12+25z$ Add 12 on each side

$15z > 22+25z$ Simplify

$15z-25z > 22+25z-25z$ Subtract 25z on each side

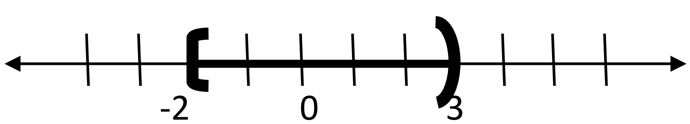
$-10z > 22$ Simplify

$\dfrac{-10z}{-10} < \dfrac{22}{-10}$ Divide -10 on each side. **Reverse** the inequality symbol.

$z < -\dfrac{11}{5}$ Simplify

The solution set is $\left\{ z \,\middle|\, z < -\dfrac{11}{5} \right\}$ Place the solution set in the set-builder notation

Interval Notation



$$[-2,3)$$

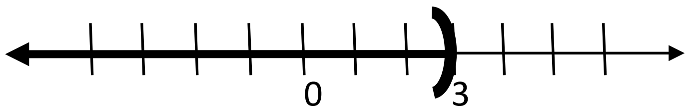
Use the open parentheses ( ) if the value is not included in the graph, i.e. greater than (>) or less than (<). Use the brackets [ ] if the value is part of the graph, i.e. greater than or equal to ($\geq$). Whenever there is a break in the graph, write the interval up to the point. Then write another interval for the section of the graph after that part. Put a union sign “$\cup$” between each interval to "join" them together.

Solve Compound Inequalities (two inequalities joined by “and” or “or”)

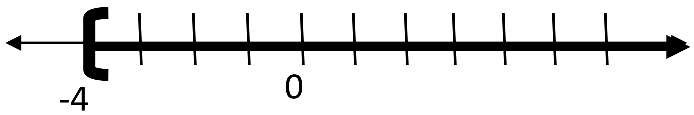
Example (1): $x < 3$ **and** $x \geq -4$

Solution: When solving compound inequalities, we usually graph them on the number lines to get the solution set.

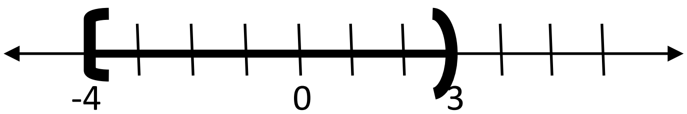
$x < 3$



$x \geq -4$



$x < 3$ **and** $x \geq -4$



* When two inequalities joined by “and”, that means **interception** of the solutions.

* Graph the inequalities separately.

* Look for overlapping of the graph.

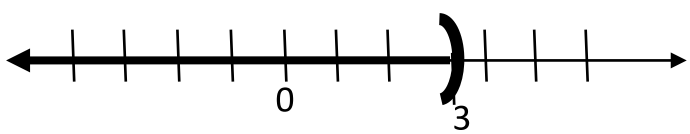
* What you see is what you get. Write out the interval notation from the overlapping segment, if any.

Interval Notation: $[-4,3)$

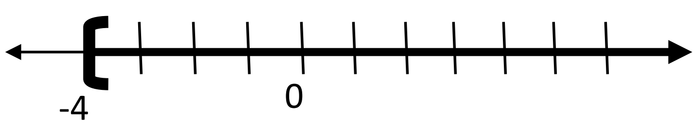
Example (2): $x < 3$ **or** $x \geq -4$

Solution: When solving compound inequalities, we usually graph them on the number lines to get the solution set.

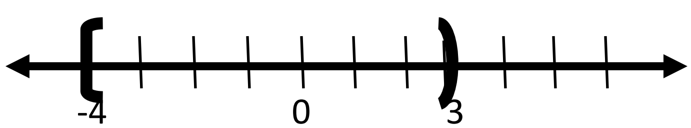
$x < 3$



$x \geq -4$



$x < 3$ **or** $x \geq -4$



When two inequalities joined by “or”, that means **union** of the solutions.

* Graph the inequalities separately.

* Look for everything shaded on the graph.

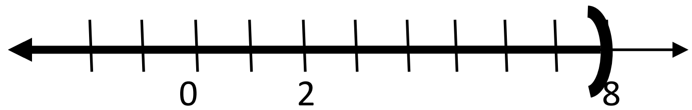
* What you see is what you get. Write out the interval notation from the number line.

Interval Notation: $(-\infty, \infty)$

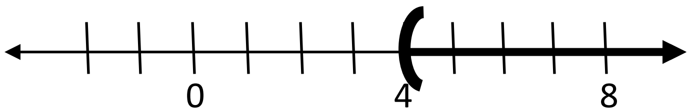
Example (3): $x+1<9$ **and** $2x-1>7$

Solution: We need to solve each inequality before we can place them on the number lines.

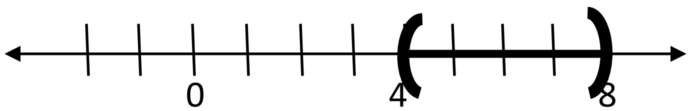
$$x+1-1<9-1$$
$$x<8$$



$$2x-1+1>7+1$$
$$2x>8$$
$$x>4$$



$x<8$ **and** $x>4$



* When two inequalities joined by "and", that means **interception** of the solutions.

* Graph the inequalities separately.

* Look for overlapping of the graph.

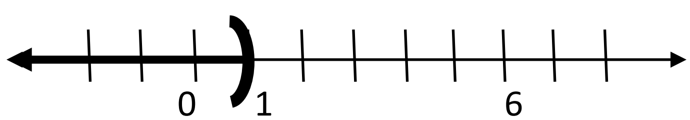
* What you see is what you get. Write out the interval notation from the overlapping segment, if any.

Interval Notation: $(4,8)$

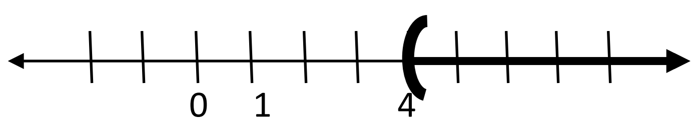
Example (4): $x+1<2$ **or** $2x-1>8$

Solution: We need to solve each inequality before we can place them on the number lines.

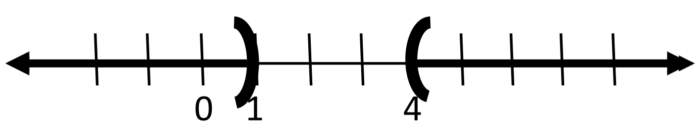
$$x+1-1<2-1$$
$$x<1$$



$$x>4$$



$x<1$ **or** $x>4$



* When two inequalities joined by "or", that means **union** of the solutions.

* Graph the inequalities separately.

* Look for everything shaded on the graph.

* What you see is what you get. Write out the interval notation from

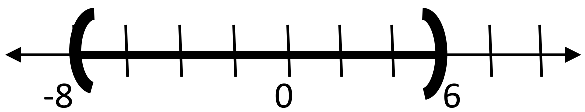
Interval Notation: $(-\infty,1)\cup(4,\infty)$

Example (5): $-5 < x + 3 < 9$

Solution: This is a **three-part** inequality. We will solve this inequality a little different than previous examples. However, our goal is to isolate the variable x in the middle.

$$-5 - 3 < x + 3 - 3 < 9 - 3$$

$$-8 < x < 6$$



$$(-8, 6)$$

*To isolate the variable x, we need to subtract 3 in the middle as well as two sides.

*State the solution in interval notation. (you can graph the solution on the number line to help you write out the interval notation.)

Example (6): $-2 < 7 - 3x \le 19$

Solution: This is a **three-part** inequality, so our goal is to isolate the variable x in the middle.

$$-2 - 7 < 7 - 7 - 3x \le 19 - 7$$

$$-9 < -3x \le 12$$

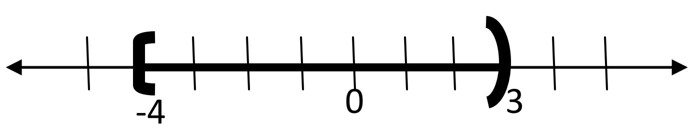
$$\frac{-9}{-3} > \frac{-3x}{-3} \ge \frac{12}{-3}$$

$$3 > x \ge -4$$

*The first thing we need to do to isolate the variable x is subtracting 7 in the middle as well as two sides.

*Next we need to divide -3 in the middle as well as two sides and **Reverse** the inequality symbol.

* State the solution in interval notation. (you can graph the solution to help you write out the interval notation.)



$$[-4, 3)$$

Exercises: Solve the following inequalities. Write the solution in interval notation.

1. $2x+1\le -1$ **or** $2x+1\ge 3$

2. $-1<5-2x\le 11$

3. $2t-3\ge 5t-(2t+1)$

4. $\dfrac{3x-2}{4}<\dfrac{2x+1}{5}$

5. $\dfrac{3}{2}(1-x)\le \dfrac{1}{4}-x$

Answers:

1. $(-\infty,-1]\cup[1,\infty)$ **2.** $[-3,3)$ **3.** $(-\infty,-2]$ **4.** $(-\infty,2)$ **5.** $\left[\dfrac{5}{2},\infty\right)$