Beyond the Textbook: Getting Developmental Mathematics Students Involved in Learning

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The Need for Change

Many college mathematics classes are taught in a traditional manner, using traditional examples from traditional textbooks. Perhaps the instructor begins by answering questions about the homework and then lectures on new material. The instructor shows various examples worked out on the board, while the students dutifully copy down these examples. In some classes, students are given the chance to try to work similar problems on their own if time permits. Students are then expected to practice these new skills by doing homework, usually problems closely resembling the examples the instructor worked in class. This homework may even be done online, with help and hints, but the problems still mimic the skills that the instructor demonstrated.

But how much learning is going on? Are students learning to merely parrot what the instructor or computer demonstrates, or do they truly understand concepts and develop the ability to apply their new knowledge to real-life situations? The *NCTM Principles and Standards for School Mathematics* document (2000) clearly states that the curriculum should "support student investigations" and "help students focus on decision making, reflection, reasoning, and problem-solving" (p. 24).

An earlier, but still important document, *Everybody Counts: A Report to the Nation on the Future of Mathematics Education* (1989), also speaks to the need for students to be involved in their learning:

> Educational research offers compelling evidence that students learn mathematics well only when they construct their own mathematical understanding. To understand what they learn, they must enact for themselves verbs that permeate the mathematics curriculum: 'examine,' 'represent,' 'transform,' 'solve,' 'apply,' 'prove,' 'communicate.' This happens most readily when students work in groups, engage in discussion, make presentations, and in other ways take charge of their own learning (p. 58).

The Executive Summary of the Mathematical Association of America's *Undergraduate Programs and Courses in the Mathematical Sciences: CUPM Curriculum Guide* (2004), seems to concur. "Every course should incorporate activities that will help all students' progress in developing analytical, critical reasoning, problem-solving, and communication skills and acquiring habits of minds" (p. 1).

AMATYC's *Beyond Crossroads* (2006) also weighs in on the subject. "Faculty will design and implement instructional activities that actively engage students in the learning of mathematics. Faculty actions [include] providing opportunities for and encouraging students to think, reflect, discuss, and write about mathematical ideas and concepts" (p. 55).

Textbook Limitations

Traditionally, developmental mathematics textbooks have been near clones of one another. The format of each section usually follows along the lines of first, a Rule in an orange box (or tan or blue or…), followed by Examples illustrating the Rule, often quite a few of them. Then there are the "naked" Exercises, perhaps 60 to 100 of them in a given section, all similar to the examples (and usually in the same order), and finally some Application Problems, maybe 10 to 12, often contrived. A few of the newer textbooks have also added a few writing prompts or perhaps a "group activity" at the end of the exercises. This is certainly a step in the right direction, but does not sufficiently overcome the traditional path of most textbooks.

Mathematics is supposed to make sense, but how can students believe that when they see application problems such as these in their textbooks?

> *9 times the length of a swimming pool is the same as 6 times the length plus 150 meters. Find the length of the pool.* [Wouldn't the author have to already know the length in order to pose this problem? So why would a student need to find that length?]

> *Charlie is in a desperate hurry to make his flight at the San Francisco airport. He accidentally rushes onto the moving walkway that is traveling opposite to the direction he is running. Find his speed through the airport if the walkway travels at a rate of 4 miles per*

hour and Charlie runs at a rate of 6 miles per hour. [Wouldn't Charlie just get off the moving walkway?]

Joe has 17 coins in his pocket, all dimes and quarters. The total value is \$2.75. How many coins of each type does he have? [Wouldn't Joe just take the coins out of his pocket and count them rather than writing a system of equations to solve the problem?]

Problems like this give math a bad name! It is no wonder that students dislike math and think it has no relevance to their lives if all they see are such "story problems," as they call them.

What Can Be Done?

Students need to be presented with authentic application problems, especially ones where they can synthesize their knowledge, apply a variety of concepts to solve the problem, and work in groups to discuss the mathematical issues involved. Coming up with such problems can be a bit daunting, especially if one starts searching the Internet for ideas. Below is a summary of some problems, called explorations, that have been used successfully in developmental mathematics classes and might be a basis for ideas for others.

Explorations Dealing with Social Issues

Garbage Exploration (Prealgebra or Elementary Algebra)

Year	Years since 1960	Municipal Solid Waste (million tons)
1960	0	88
1970	10	121
1980		152
1990		197
	34	209
2000		239
2004	44	250
2005		253
	46	254
2007		255
	50	250

http://www.epa.gov/wastes/nonhaz/municipal/pubs/2010_MSW_Tables_and_Figures_508.pdf (p. 2, table 1)

http://www.epa.gov/osw/nonhaz/municipal/pubs/msw07-rpt.pdf (p. 11, table ES-1)

Complete the table (x = Years since 1960, y = Municipal Waste in millions of tons).

Create a scatterplot (Students decide on appropriate scales to best display the data).

Fit a line to the data by eye.

Choose two points on the line.

Write the equation of the line.

What is the slope? In a sentence, explain what the slope means about the amount of municipal waste.

What is the y-intercept? In a sentence, explain what the y-intercept means about the amount of municipal waste.

Use the model to predict waste for given years and/or the year when waste will reach a given amount.

World Population (Intermediate Algebra: Uses a graphing calculator for regression equations)

Given Data:

Year	1960	1970	1980	1990	2000
Pop. in Billions	3.04	3.692	4.447	5.296	6.118

http://www.google.com/publicdata/explore?ds=d5bncppjof8f9_&met_y=sp_pop_totl&tdim=true&dl=en&hl=en&q=world+population

Assume the world population is growing exponentially. Let $E(t)$ represent the world population since 1900. Find the equation for $E(t)$.

Assume the world population is growing linearly. Let $L(t)$ represent the world population since 1900. Find the equation for $L(t)$.

Assume the world population is experiencing quadratic growth. Let $Q(t)$ represent the world population since 1900. Find the equation for $Q(t)$.

Complete a table for population in billions from 1880 to 2050, using all three models.

Graph all three models on the same axes, choosing appropriate scales.

Calculate and explain in complete sentences what your answers mean about the population. For example, $L(112)$, $E(112)$, $Q(112)$, $L^{-1}(0)$, $E^{-1}(0)$, vertex of $Q(t)$, or others.

Rank the functions and explain your reasoning with respect to

- modeling the given data (1960–2000);
- predicting the population for 2011 (actual population 6.974 billion);
- predicting the population for 2150.

Explorations Dealing with Everyday Issues

Appliance Costs (Prealgebra or Elementary Algebra) Sample only: Obtain current prices

Brand	Size	Initial Cost	Estimated Annual Cost (at 6 cents/kwh)
Amana	21 ft^3	\$875	\$34
Whirlpool	21 ft^3	\$800	\$39
Tappan	21 ft^3	\$650	\$57
Kitchen Aid	21 ft^3	\$950	\$31

For each refrigerator, write an equation that describes the relationship between expected life, x, in years and total cost, C.

Complete a table for each refrigerator's total costs for 0 to 25 years (every 5 years).

Find when the total costs of *Whirlpool* and *Tappan* are the same.

Find when the cost of *Kitchen Aid* is less than *Amana*.

Graph all four lines on the same graph (optional)

Write a letter to the Costanza's, who are shopping for a new refrigerator. Include possible years of ownership and recommended brands to buy for the lowest costs.

Credit Card Debt (Elementary Algebra)

Read the story below, then complete the table to show the amount owed at the end of each month:

Muffy saw an absolutely gorgeous necklace on sale after Christmas that she felt that she must have. She had just received a new credit card that had an APY of 14.4% (monthly interest of 1.2%) and required only a minimum payment of \$40 a month. Muffy decided to spend the \$3000 for the necklace as she felt that she could easily make the minimum payments. For the first six months, she made every payment on time. In July, Muffy became sick and could not make the payments. The credit card company was magnanimous and said that she could wait 6 months before resuming her monthly payments although the interest would continue. Fill out the table below to see how much Muffy still owed in July and how much she will owe at the end of the year.

What's the moral?

Saving (Elementary or Intermediate Algebra)

Read the story below:

Sara and Sheila are twins who began to work at age 20 with identical jobs and starting salaries. At the beginning of each year starting at the age of 30, they received identical bonuses of \$2500. In other ways, the twins were not identical. Early in life, Sara was conservative. Each year, she invested the \$2500 bonus in a savings program earning 8% interest compounded annually. At age 40, Sara decided to have some fun and began spending her \$2500 bonus, but she let her earlier investment continue to earn interest. This continued until she was 65. In contrast, for the first 10 years she worked, Sheila spent her \$2500 bonuses. At age 40, she began to invest her bonus every year in the same kind of account (8% interest compounded annually). This continued until Sheila was 65 years old.

Complete tables from age 20 to 65 for both women:

Deposit on 1/1 of each year	Total at Year End Before Interest	Total at Year End Including Interest

Find totals deposited and totals in the savings accounts at age 65.

Discuss advantages for Sara's plan, advantages for Sheila's plan.

What's the moral?

Car Depreciation (Intermediate Algebra)

Let p represent the advertised price (in dollars) of your car when it is t years old (based on the current year of 2012). Do not use the actual year as in 2003. A 2003 car would be listed as 9 years old.

Month	Amount owed at beginning of month	Interest	Amount Owed + Interest	Payment	Owed at end of month

Table for "Credit Card Debt" exercise

Sample Table (provide tables for several makes of cars). This data was found on July 23, 2012. Update with a local newspaper using current prices.

Ford Explorer Model year	t Age of car in 2012	p Price of car
'06		$10,977
'05		$10,977
'04		$9,977
'04		$11,477
'04		$9,977
'04		$7,595
'03		$3,977
'02		$8,977
'02		$6,895
'99		$2,977

Complete the table: Age of car and Price.

Create a scatterplot.

Sketch a line of best fit (by eye).

From the graph, find p when $t = 5$.

From the graph, find t when $p = \$8,000$.

Identify two points on the line.

Write the equation of the line, $p = f(t)$.

Use regression feature of the calculator to calculate the line of best fit, $p = g(t)$.

- What is the slope of g, and what does it mean?
- What is the p-intercept of g, and what does it mean?
- What is the t-intercept of g, and what does it mean?
- Calculate $g(2)$ and explain what it means.
- Calculate t when $g(t) = 8000$ and explain what it means.
- Do you think g models the car situation well near the p-intercept? Why or why not?
- Do you think g models the car situation well near the t-intercept? Why or why not?

Note: All these questions could be asked for $f(t)$ instead or in addition to $g(t)$, if a graphing calculator is not available.

Explorations Dealing with Other Interests (Summaries only)

Percent of Fat (Prealgebra)

- Use fast food menus. Students might be asked to gather these in preparation for this exploration.
- Choose 3 meals, at least 9 items.
- Total calories should be between 1500 and 3000.
- Total fat should be less than 30%.
- Complete a table (*Item, Calories, Grams of fat, Calories from fat, % calories*)

Classic Cars (Elementary or Intermediate Algebra)

- Start with the value of a new car.
- Exponential decay for 15 years.
- Constant value for 5 years.
- Exponential growth (as collector's item).
- Create a table.
- Create a graph.
- Calculations and Interpretative Questions.

Speed/Triathlon (Elementary Algebra)

- A piecewise function graph is given.
- Write a paragraph to explain the story.
- Triathlon Story (real data) is given.
- Create a table.
- Create a graph.

Murder Mystery: Who Murdered Professor Euler? (Intermediate Algebra)

- The body is found! Clues are provided.
- Find regression equations (linear and exponential).
- Make the graphs.
- The scene is set.
- The murderer is apprehended!

Summary

There are many sources of authentic data from which inte esting problems can be derived. A search on the Internet ca provide data for many possibilities. If one can arrange to hav a classroom where students sit in groups at tables, rather tha lined up in rows expecting a sage-on-the-stage, the ability have students discuss problems such as those suggested here certainly enhanced. While students are in groups, they can also discuss problems from the text or interesting worksheets that the instructor might provide. Current events can also lead to interesting problems for consideration.

Richard Skemp (1976), in his classic article, *Relational Understanding and Instrumental Understanding*, suggests that two very different subjects are being taught under the name of mathematics. Developmental mathematics classes have traditionally leaned more heavily toward what Skemp refers to as "instrumental understanding" — learning the rules, memorizing the procedures, and parroting them back to the instructor. Incorporating more authentic problems in the curriculum, especially ones where students must apply their knowledge and interpret their results, can help move the curriculum more towards the relational understanding that Skemp supports. In the twenty-first century, students and instructors need to move beyond the textbook.

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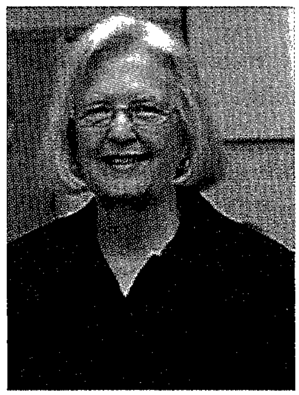

Janet Teeguarden, Professor of Mathematics at Ivy Tech Community College, was awarded both a NISOD Excellence Award and an AMATYC Teaching Excellence Award in 2011. She has taught at Ivy Tech since 2000, following 20 years on the faculty at DePauw University. Janet has taught all levels of undergraduate mathematics, but has a special love for the developmental mathematics she currently teaches. She is a recent department chair, former president of InMATYC, and former secretary and board member of ICTM. She is a frequent presenter at local, state, and national conferences and serves as a reviewer for the *Math*AMATYC *Educator*.

Lucky Larry

The following are two ways of solving the same equation, as obtained from two creative students:

Solve
$$\begin{aligned} 4-(x-1) &= 7+5x \\ -4x+4 &= 7+5x \\ -4x+4-7 &= 7+5x-7 \\ -4x-3 &= 5x \\ -4x-3+4x &= 5x+4x \\ -3 &= 9x \\ x &= -\frac{1}{3} \end{aligned}$$

Solve
$$\begin{aligned} 4-(x-1) &= 7+5x \\ 4-x-1 &= 7+5x \\ 5-x &= 7+5x \\ 5-x-7 &= 7+5x-7 \\ -x-2 &= 5x \\ -x-2+x &= 5x+x \\ -2 &= 6x \\ x &= -\frac{1}{3} \end{aligned}$$

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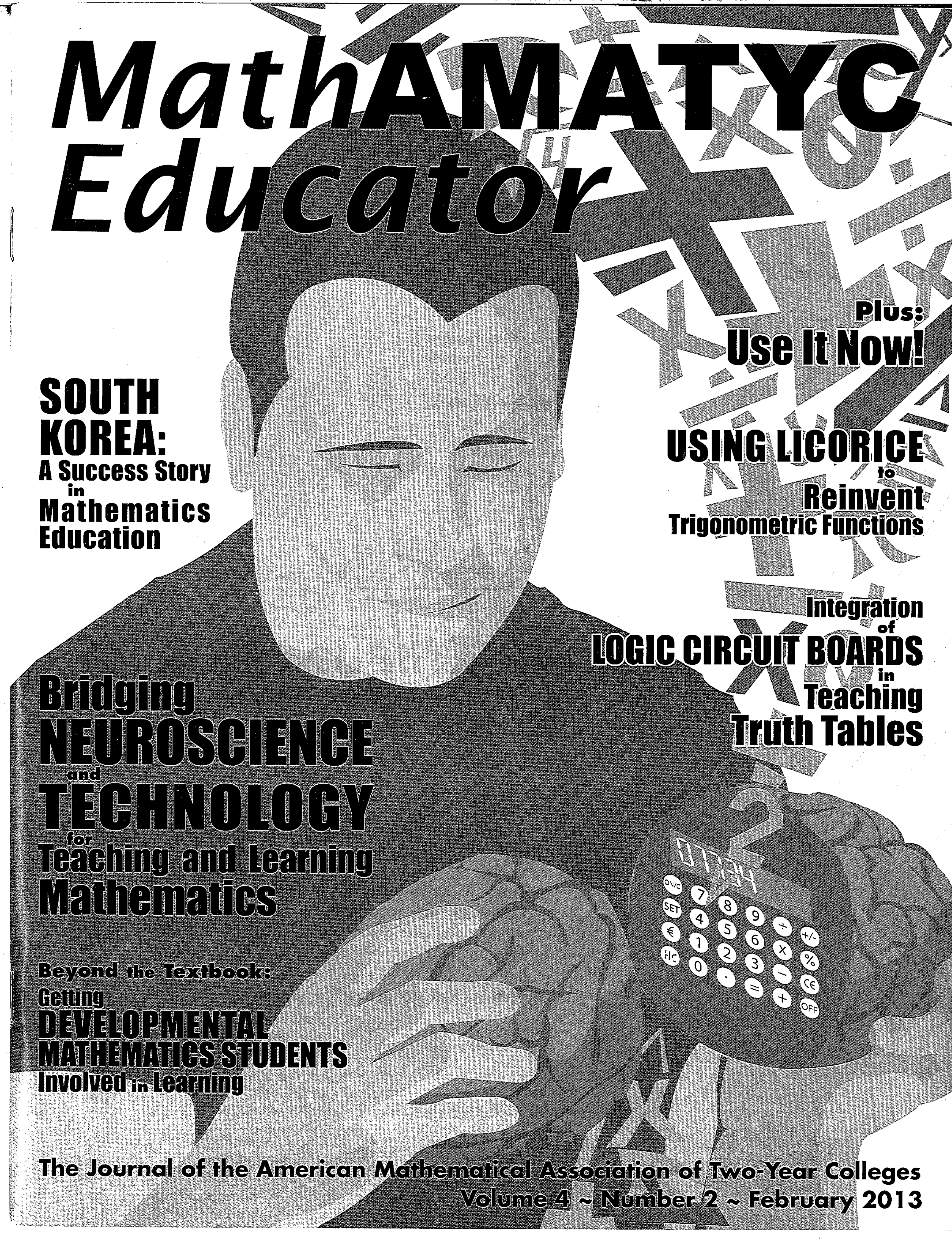

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